

ADHESION AND THE GEOMETRY OF THE COSMIC WEB

JOHAN HIDDING*, RIEN VAN DE WEYGAERT, GERT VEGTER
and BERNARD J.T. JONES

*Kapteyn Astronomical Institute, University of Groningen,
Groningen, 9747 AD, The Netherlands*

**E-mail: hidding@astro.rug.nl
www.rug.nl/sterrenkunde*

We present a new way to formulate the geometry of the Cosmic Web in terms of Lagrangian space. The Adhesion model has an ingenious geometric interpretation out of which the spine of the Cosmic Web emerges naturally. Within this context we demonstrate a deep connection of the relation between Eulerian and Lagrangian space with that between Voronoi and Delaunay tessellations.

Keywords: Cosmology; large-scale structure of universe; cosmic web; adhesion model

1. Introduction

The Cosmic Web is the largest known structure in the Universe. It is seen in both observations¹ and simulations.² The Adhesion model^{3–5} provides a heuristic model describing both the intricate geometry of the Cosmic Web, and the non-local nature of its dynamics. Before shell crossing occurs, Adhesion follows the Zel'dovich approximation⁶

$$\mathbf{x}(\mathbf{q}, D_+) = \mathbf{q} - D_+ \nabla_{\mathbf{q}} \Phi_0(\mathbf{q}), \quad (1)$$

describing the motion of a particle labeled \mathbf{q} in comoving coordinates as having a constant comoving velocity $\mathbf{v} = \partial \mathbf{x} / \partial D_+ = -\nabla_{\mathbf{q}} \Phi$. The Adhesion model adds a viscosity term to this recipe, emulating the adhesive effects of gravity, preventing shell-crossing from ever taking place. The resulting equation of motion is known as Burgers' equation

$$\frac{\partial \mathbf{v}}{\partial D_+} + (\mathbf{v} \cdot \nabla_{\mathbf{x}}) \mathbf{v} = \nu \nabla_{\mathbf{x}}^2 \mathbf{v}. \quad (2)$$

In the limit where $\nu \rightarrow 0$, it has the exact solution⁷

$$\Phi(\mathbf{x}, D_+) = \max_{\mathbf{q}} \left[\Phi_0(\mathbf{q}) - \frac{(\mathbf{x} - \mathbf{q})^2}{2D_+} \right]. \quad (3)$$

This solution gives us not only the potential at some growing-mode time D_+ , but also the mapping from Eulerian coordinates $\mathbf{x} \in \mathcal{E}$ to Lagrangian coordinates $\mathbf{q} \in \mathcal{L}$, as the Lagrangian coordinate where the maximum is attained.

2. Weighted Voronoi diagrams

We found expression (3) to be identical to that of the *weighted Voronoi diagram* of the set of points \mathbf{q} , weighted by the value of the velocity potential.⁸ The weighted

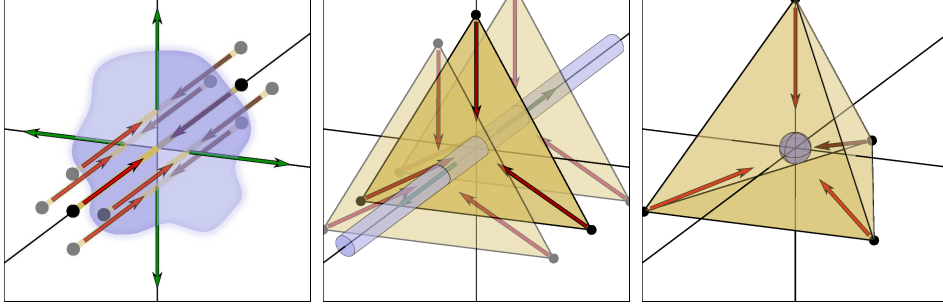


Fig. 1. The duality of three morphological types: walls, filaments and clusters. Each time the Eulerian structure is shown in blue, and the Lagrangian mass building the structure in yellow. The green arrows show the motion internal to the structure. A wall is a flattened structure, where locally the mass originates from a line-like region. A filament is elongated, however each part of the filament was originally distributed as a sheet. The cluster has collapsed from three directions.

Voronoi cell of a point $\mathbf{q} \in \mathcal{L}$ is given by

$$V_q = \left\{ \mathbf{x} \in \mathcal{E} \mid (\mathbf{x} - \mathbf{q})^2 + w_q \leq (\mathbf{x} - \mathbf{p})^2 + w_p, \mathbf{p} \in \mathcal{L} \right\}. \quad (4)$$

The physical interpretation of this expression is that the Voronoi cell gives us the Eulerian region of space a parcel of matter occupies. If we invert this relation, we find that the Delaunay triangulation, being the dual of the Voronoi diagram tells us where the matter around an Eulerian location came from. Describing cosmic structures in this way, allows us to trace their formation and change in topology over time.

3. Evolution of the Cosmic Web

We can find the walls of the web structure as edge-like objects in Lagrangian space, whereas filaments have a flattened signature. Clusters, being the most massive concentrations of matter are therefore most extended in Lagrangian space. Looking at figure 2, we see voids growing to compress their weaker neighbours, while their area in Lagrangian space shrinks as matter flows into the walls and filaments bounding the voids.

References

1. V. de Lapparent, M. J. Geller and J. P. Huchra, *Astrophysical Journal* **302**, L1 (1986).
2. V. Springel et al., *Nature* **435**, 629 (2005).
3. S. N. Gurbatov and A. I. Saichev, *Radiophysics & Quantum Electronics* **27**, 303 (1984).
4. S. F. Shandarin and Y. B. Zel'dovich, *Reviews of Modern Physics* **61**, 185 (1989).
5. M. Vergassola et al., *Astronomy & Astrophysics* **289**, 325 (1994).
6. Y. B. Zel'dovich, *Astronomy & Astrophysics* **5**, 84 (1970).
7. E. Hopf, *Communications on Pure and Applied Mathematics* **Vol. 3, No. 3**, 201 (1950).
8. J. Hidding, R. van de Weygaert, G. Vegter and B. J. T. Jones, *to be subm.* (2012).

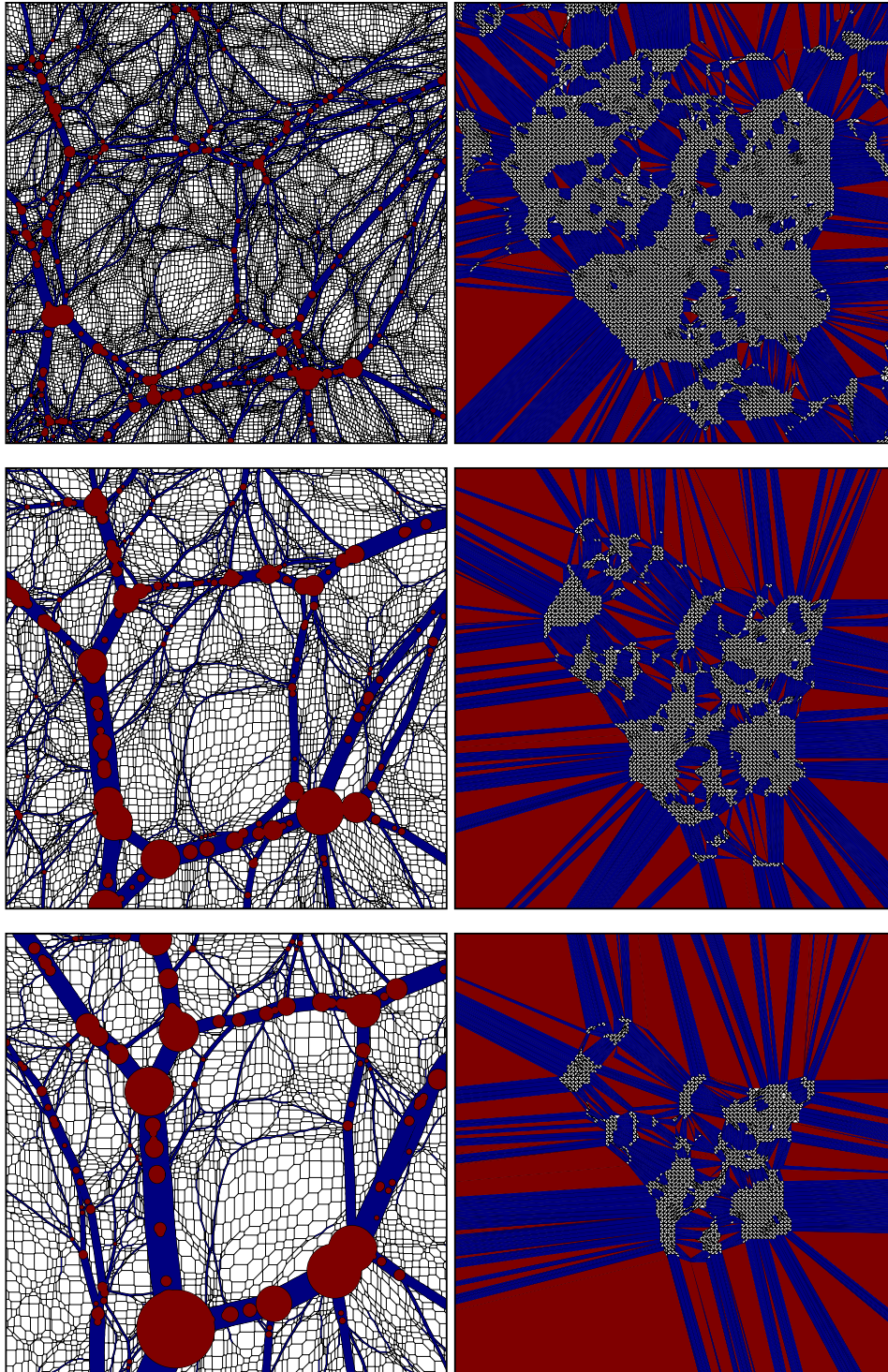


Fig. 2. Time evolution in Eulerian (left) and Lagrangian (right) space of a 2-D model. On the right we see the Lagrangian areas corresponding to the clusters (red triangles), and filaments (blue lenticular regions) shown on the left. White is for voids.